In [63]:

```
import numpy as np
from numpy import cov,corrcoef
import matplotlib.pyplot as plt
import astropy.units as u
from scipy.optimize import curve_fit
import scipy
import scipy.stats as stats
import pandas as pd
import sklearn
from sklearn import linear_model
from sklearn.linear_model import LinearRegression

ledd=np.log10(1.3)+38#Eddington luminosity of Solar Mass

#%matplotlib inline
```

In [93]:

```
def linear(x,a,b,c):
    return a+b*x[0]+c*x[1]
```

In [30]:

```
rqa=np.loadtxt('Downloads/fundamental_data/data0612radioquiet.txt')#radio quiet agn
xrb=np.loadtxt('Downloads/fundamental_data/data0612allxrb.txt')#all xrb data
anx=np.loadtxt('Downloads/fundamental_data/data0612allagnandxrb.txt')#all data
rla=np.loadtxt('Downloads/fundamental_data/data0612radioloud.txt')#all radio loud a
rqx=np.loadtxt('Downloads/fundamental_data/data0612radioquietagnandxrb.txt')#all ra
agn=np.loadtxt('Downloads/fundamental_data/data0612allagn.txt')
```

In [56]:

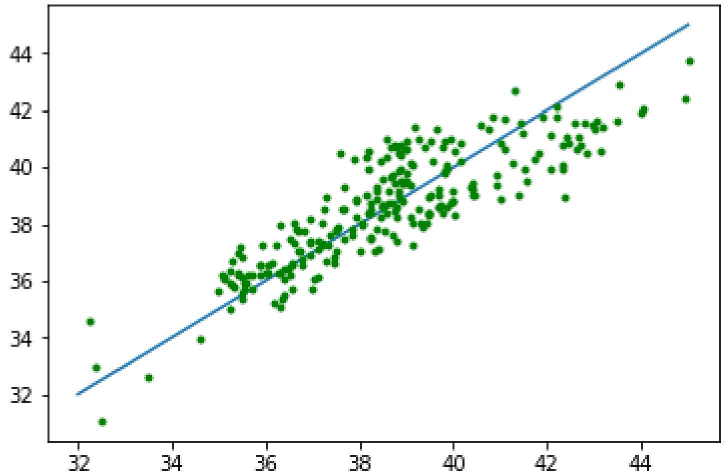
```
```

In [ ]:

```
```

In [99]:

```
#all agn
#curve_fit
x0=agn[:,1:3]
y=agn[:,0]
x=np.transpose(x0)
popt,pcov=curve_fit(linear,x,y)
perr=np.sqrt(np.diag(pcov))
xx=np.arange(32,46)
yy=np.arange(32,46)
plt.plot(xx,yy,'-')
plt.plot(y,linear(x,*popt),'g.')
plt.show()
print(popt)
print(perr)
```

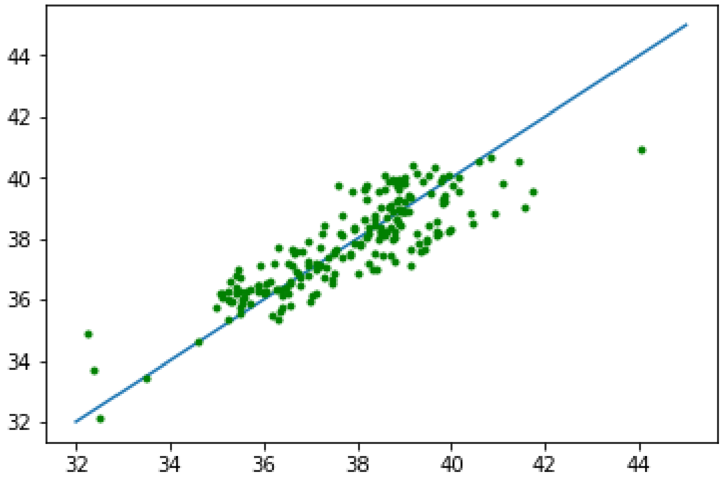



```
[ 1.35775761  0.71921014  0.89322117]
[ 1.35202488  0.03082648  0.08722038]
```

#same with matrix method [[1.35775777] [0.71921014] [0.89322117]]

In [100]:

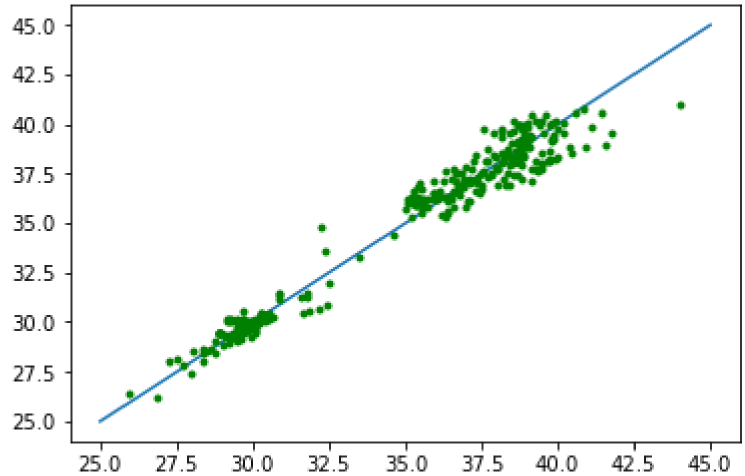
```
#radio quiet agn
x0=rqa[:,1:3]
y=rqa[:,0]
x=np.transpose(x0)
popt,pcov=curve_fit(linear,x,y)
perr=np.sqrt(np.diag(pcov))
xx=np.arange(32,46)
yy=np.arange(32,46)
plt.plot(xx,yy,'-')
plt.plot(y,linear(x,*popt),'g.')
plt.show()
print(popt)
print(perr)
```




```
[ 8.19968715   0.58957793   0.66442534]
[ 1.2811166    0.02776613   0.07766573]
```

In [101]:

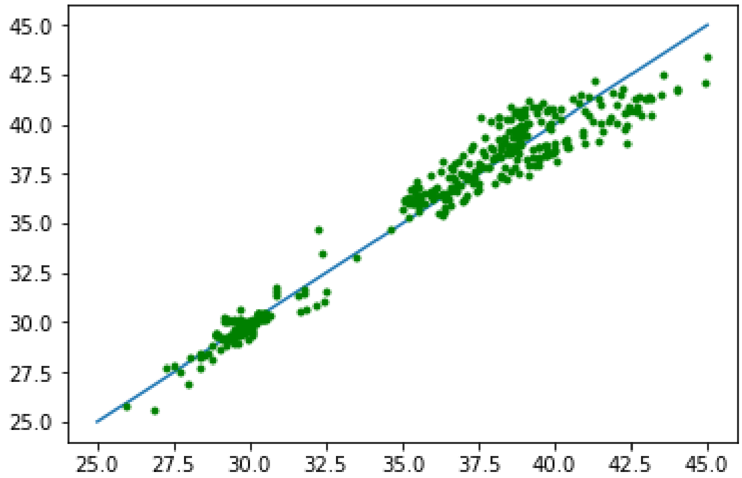
```
#radio quiet agn and xrb
x0=rqx[:,1:3]
y=rqx[:,0]
x=np.transpose(x0)
popt,pcov=curve_fit(linear,x,y)
perr=np.sqrt(np.diag(pcov))
xx=np.arange(25,46)
yy=np.arange(25,46)
plt.plot(xx,yy,'-')
plt.plot(y,linear(x,*popt),'g.')
plt.show()
print(popt)
print(perr)
```




```
[ 7.59597056  0.59542602  0.70983916]
[ 0.72176607  0.02040482  0.01987654]
```

In [102]:

```
#agn and xrb
x0=anx[:,1:3]
y=anx[:,0]
x=np.transpose(x0)
popt,pcov=curve_fit(linear,x,y)
perr=np.sqrt(np.diag(pcov))
xx=np.arange(25,46)
yy=np.arange(25,46)
plt.plot(xx,yy,'-')
plt.plot(y,linear(x,*popt),'g.')
plt.show()
print(popt)
print(perr)
```

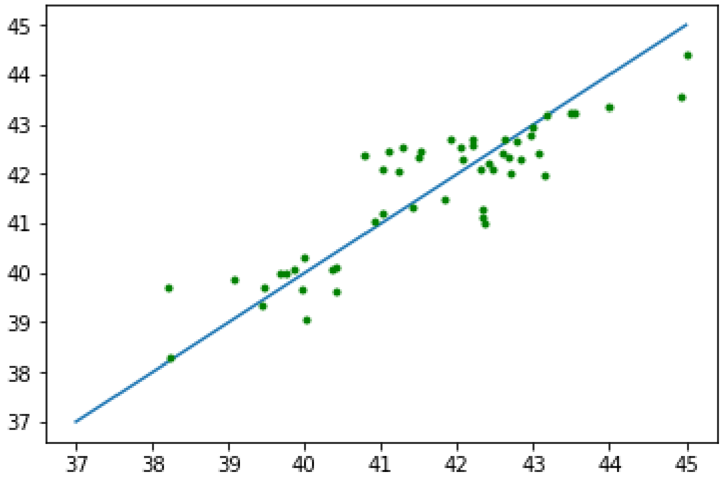



```
[ 3.95561574  0.69602065  0.69310598]
[ 0.85187042  0.02412848  0.02479418]
```

In [104]:

```
#radio loud agn
x0=rla[:,1:3]
y=rla[:,0]

x=np.transpose(x0)
popt,pcov=curve_fit(linear,x,y)
perr=np.sqrt(np.diag(pcov))
xx=np.arange(37,46)
yy=np.arange(37,46)
plt.plot(xx,yy,'-')
plt.plot(y,linear(x,*popt),'g.')
plt.show()
print(popt)
print(perr)
```




```
[ 7.68779659  0.79159666 -0.06076855]
[ 2.82630251  0.05964385  0.18069606]
```

In [242]:

```
#radio loud agn
x=rla[:,1]
y=rla[:,0]
fit=np.polyfit(x,y,1)
print(fit)
np.corrcoef(x,y)
```

```
[ 0.78859975  7.29301251]
```

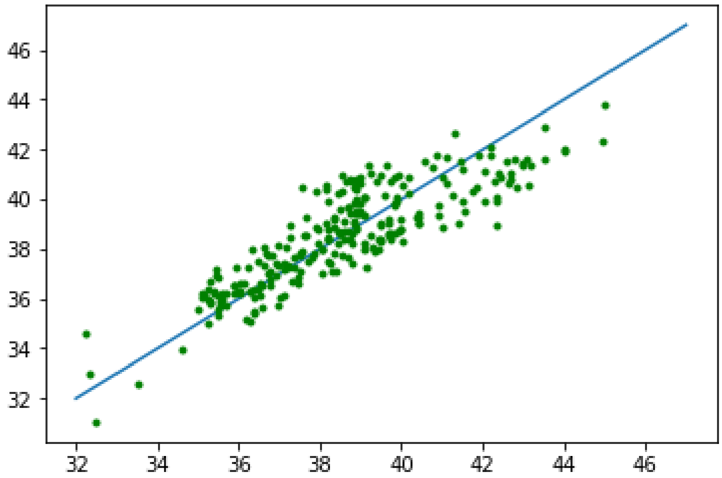
Out[242]:

```
array([[ 1.        ,  0.88572957],
       [ 0.88572957,  1.        ]])
```

In [76]:

```
#matrix method
agn=np.loadtxt('Downloads/fundamental_data/data0612allagn.txt')
agn1=np.loadtxt('Downloads/fundamental_data/data0612allagn.txt')
x0=agn[:,0:3]
x0[:,0]=1
y0=agn1[:,0]
x=np.matrix(x0)
xt=x.T
y=np.matrix(y0)
yt=y.T
theta=(xt*x).I*xt*yt
print(theta)
ypred=theta[0]*x0[:,0]+theta[1]*x0[:,1]+theta[2]*x0[:,2]
ypred.shape,y.shape
xx=np.arange(32,48)
yy=np.arange(32,48)
plt.plot(xx,yy,'-')
plt.plot(y,ypred,'g.')
plt.show()
```

```
[[ 1.35775777]
 [ 0.71921014]
 [ 0.89322117]]
```



In [ ]:

In [ ]:

In [ ]:

In [ ]:

In [ ]:

```

```

In [ ]:

```

```

In [ ]:

```

```

In [ ]:

```

```

In [ ]:

```

```

###methods test

In [137]:

```python
import statsmodels.api as sm
import numpy as np

def regress_m(y,x):
    ones=np.ones(len(y))
    X=sm.add_constant(np.column_stack((x[0],ones)))
    for ele in x[1:]:
        X=sm.add_constant(np.column_stack((ele,X)))
    results=sm.OLS(y,X).fit()
    return results

#test statsmodel model

x=np.random.rand(2,100)*10+20
y=3+0.6*x[0]+1.1*x[1]+np.random.rand(100)*2
print(regress_m(y,x).summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:                      y   R-squared:
   0.972
Model:                            OLS   Adj. R-squared:
   0.971
Method:                 Least Squares   F-statistic:
   1687.
Date:                Tue, 13 Jun 2017   Prob (F-statistic):
 4.48e-76
Time:                        16:41:25   Log-Likelihood:
-91.507
No. Observations:                 100   AIC:
   189.0
Df Residuals:                      97   BIC:
   196.8
Df Model:                           2

Covariance Type:            nonrobust

======================================================================
========
                 coef    std err          t      P>|t|      [95.0% Con
f. Int.]
----------------------------------------------------------------------
--------
x1             1.0949      0.021     51.851      0.000         1.053
   1.137
x2             0.5833      0.020     29.152      0.000         0.544
   0.623
const          4.7051      0.747      6.297      0.000         3.222
   6.188
======================================================================
========
Omnibus:                       32.472   Durbin-Watson:
   2.166
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
```

```
   8.329
Skew:                          -0.395   Prob(JB):
  0.0155
Kurtosis:                       1.827   Cond. No.
    430.
======================================================================
========

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

In [166]:

```
import statsmodels.api as sm
import numpy as np

nobs=100
X=np.random.random((nobs,2))
X=sm.add_constant(X)
theta=[3,0.6,1.1]
e=np.random.random(nobs)
y=np.dot(X,theta)+e

result=sm.OLS(y,X).fit()
print(result.summary())
```

```
                            OLS Regression Results                            
==============================================================================
========
Dep. Variable:                      y   R-squared:                       
   0.644
Model:                            OLS   Adj. R-squared:                  
   0.637
Method:                 Least Squares   F-statistic:                     
   87.81
Date:                Fri, 16 Jun 2017   Prob (F-statistic):           
 1.71e-22
Time:                        09:36:52   Log-Likelihood:               
-15.231
No. Observations:                 100   AIC:                             
   36.46
Df Residuals:                      97   BIC:                             
   44.28
Df Model:                           2                                         

Covariance Type:            nonrobust                                         

==============================================================================
========
                 coef    std err          t      P>|t|      [95.0% Con
f. Int.]
------------------------------------------------------------------------------
--------
const          3.3628      0.073     45.821      0.000         3.217 
   3.508
x1             0.6204      0.103      6.025      0.000         0.416 
   0.825
x2             1.2390      0.106     11.724      0.000         1.029 
   1.449
==============================================================================
========
Omnibus:                       18.752   Durbin-Watson:                   
   2.238
Prob(Omnibus):                  0.000   Jarque-Bera (JB):                
   6.323
Skew:                           0.327   Prob(JB):                       
  0.0424
Kurtosis:                       1.956   Cond. No.                        
    5.26
==============================================================================
```

========

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [ ]:

```

```

In [81]:

```
#test sklearn LinearRegression
x=np.random.rand(2,100)*10+20
y=3+0.6*x[0]+1.1*x[1]+np.random.rand(100)*2
x=np.transpose(x)
clf=LinearRegression()
clf.fit(x,y)
print(clf.coef_,clf.intercept_,clf.score(x,y))
```

```
[ 0.59753449  1.08251449] 4.44897482188 0.978455865471
```

In [ ]:

```

```

In [ ]:

```

```

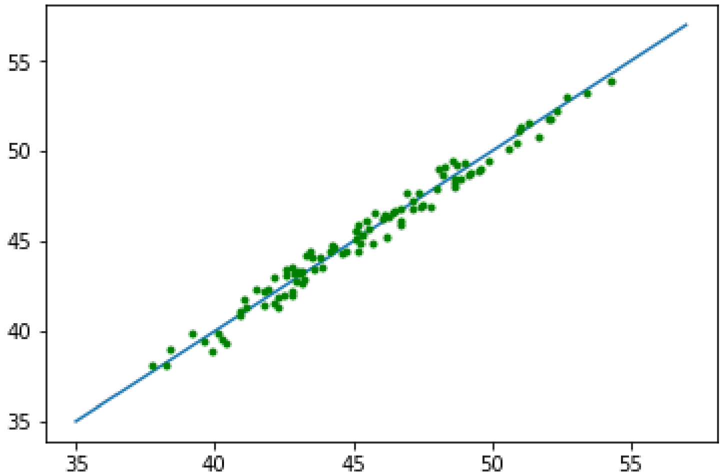
In [ ]:

```

```

In [130]:

```
#test curve_fit
x=np.random.rand(2,100)*10+20
y=3+0.6*x[0]+1.1*x[1]+np.random.rand(100)*2
popt,pcov=curve_fit(linear,x,y)
perr=np.sqrt(np.diag(pcov))
xx=np.arange(35,58)
yy=np.arange(35,58)
plt.plot(xx,yy,'-')
plt.plot(y,linear(x,*popt),'g.')
plt.show()
print(popt)
print(perr)
```




```
[ 3.9722345   0.60046732  1.09975255]
[ 0.64520401  0.01913214  0.01803415]
```

In [40]:

```
#test linear regression
import numpy as np
a=np.random.rand(100,3)*10+20
a[:,0]=1
x=np.matrix(a)

b=3+0.6*a[:,1]+1.1*a[:,2]+np.random.rand(100)*2
y=np.matrix(b)
xt=x.T
yt=y.T
theta=(xt*x).I*xt*yt
print(theta)
print(a.shape,b.shape,y.shape)
```

```
[[ 3.78874166]
 [ 0.61759225]
 [ 1.0868239 ]]
(100, 3) (100,) (1, 100)
```

In [184]:

```
import lmfit
from scipy.optimize import leastsq
import matplotlib
import matplotlib.pyplot as plt
import numpy as np

def residual(p,x,y,sigma0=0.3):
    a=p[0]
    b=p[1]
    c=p[2]
    model=a+b*x[0]+c*x[1]
    return (model-y)/sigma0

x=np.random.rand(2,100)*10+20
y=3+0.6*x[0]+1.1*x[1]+np.random.random(100)*0.3
p=[3,0.6,1.1,0.3]
out=leastsq(residual,p,args=(x,y,0.3))
print(out)
```

```
(array([ 3.1110631 ,  0.60284854,  1.09880373,  0.3        ]), 2)
```

In [ ]:

```

```

In [236]:

```
import lmfit
from scipy.optimize import leastsq
import matplotlib
import matplotlib.pyplot as plt
import numpy as np

x=np.random.rand(2,100)*10+20
y=3+0.6*x[0]+1.1*x[1]+np.random.random(100)*0.3


p=lmfit.Parameters()
p.add_many(('a',3),('b',0.6),('c',1.1))


def residual(p):
    sigma0=0.3
    a=p['a']
    b=p['b']
    c=p['c']
    model=a+b*x[0]+c*x[1]
    return (model-y)/sigma0

mini=lmfit.Minimizer(residual,p)
out1=mini.minimize(method='Nelder')
out2=mini.minimize(method='leastsq',params=out1.params)
```

In [241]:

```
from lmfit import Parameters,minimize,fit_report
print(fit_report(out2))
```

```
[[Fit Statistics]]
    # function evals   = 7
    # data points      = 100
    # variables        = 3
    chi-square         = 8.519
    reduced chi-square = 0.088
    Akaike info crit   = -240.282
    Bayesian info crit = -232.467
[[Variables]]
    a:   3.22171127 +/- 0.120359 (3.74%) (init= 3.221662)
    b:   0.59728997 +/- 0.003225 (0.54%) (init= 0.5972919)
    c:   1.09966194 +/- 0.003388 (0.31%) (init= 1.099662)
[[Correlations]] (unreported correlations are <  0.100)
    C(a, c)                      = -0.735
    C(a, b)                      = -0.692
```

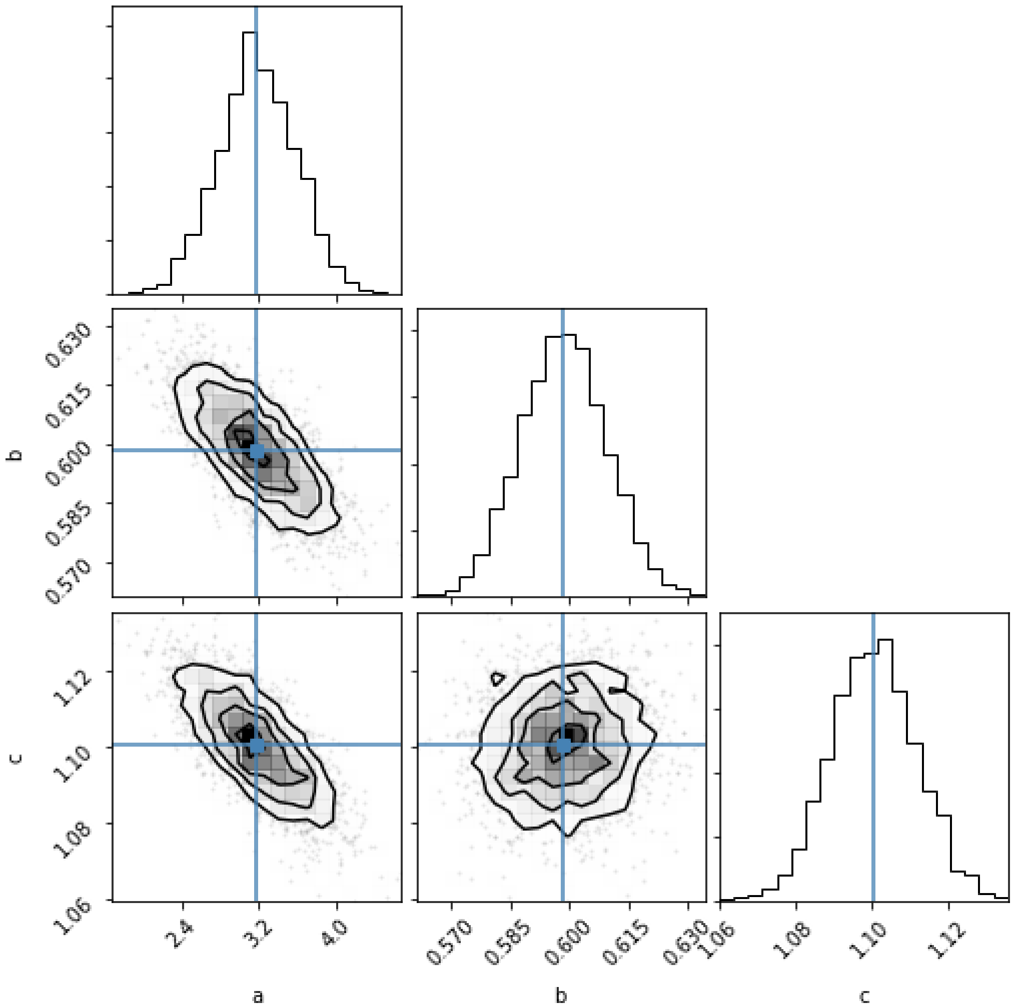
In [238]:

```
res=mini.emcee(burn=300,steps=600,thin=10,params=out1.params)
```

In [239]:

```
import corner
corner.corner(res.flatchain,labels=res.var_names,truths=list(res.params.valuesdict(
```

Out[239]:



In [ ]:

```

```

In [ ]:

```

```

In [195]:

```
lmfit.report_fit(out2.params,min_correl=0.5)
```

```
[[Variables]]
    a:   3.07543544 +/- 0.118937 (3.87%) (init= 3.075437)
    b:   0.59911359 +/- 0.003332 (0.56%) (init= 0.5991131)
    c:   1.10415315 +/- 0.003088 (0.28%) (init= 1.104153)
[[Correlations]] (unreported correlations are <  0.500)
    C(a, b)                      = -0.757
    C(a, c)                      = -0.708
```

In [200]:

```
lmfit.report_errors(out2.params,min_correl=0.5)
```

```
[[Variables]]
    a:   3.07543544 +/- 0.118937 (3.87%) (init= 3.075437)
    b:   0.59911359 +/- 0.003332 (0.56%) (init= 0.5991131)
    c:   1.10415315 +/- 0.003088 (0.28%) (init= 1.104153)
[[Correlations]] (unreported correlations are <  0.500)
    C(a, b)                      = -0.757
    C(a, c)                      = -0.708
```

In [199]:

```
lmfit.report_fit(out1.params)
```

```
[[Variables]]
    a:   3.07543691 (init= 3)
    b:   0.59911312 (init= 0.6)
    c:   1.10415327 (init= 1.1)
```

In [196]:

```
lmfit.report_fit(out1.params,min_correl=0.5)
```

```
[[Variables]]
    a:   3.07543691 (init= 3)
    b:   0.59911312 (init= 0.6)
    c:   1.10415327 (init= 1.1)
```

In [203]:

```
ci,trace=lmfit.conf_interval(mini,out2,trace=True,verbose=False)
```

In [204]:

```
lmfit.printfuncs.report_ci(ci)
```

```
      99.73%    95.45%    68.27%    _BEST_    68.27%    95.45%    99.7
3%
 a:  -0.36624  -0.24098  -0.11957   3.07544  +0.11957  +0.24098  +0.36
624
 b:  -0.01026  -0.00676  -0.00335   0.59911  +0.00335  +0.00676  +0.01
026
 c:  -0.00950  -0.00626  -0.00309   1.10415  +0.00309  +0.00626  +0.00
950
```

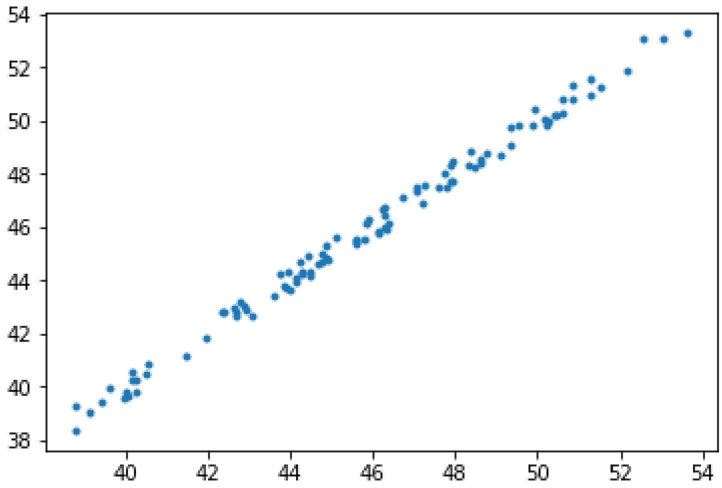
In [ ]:

```

```

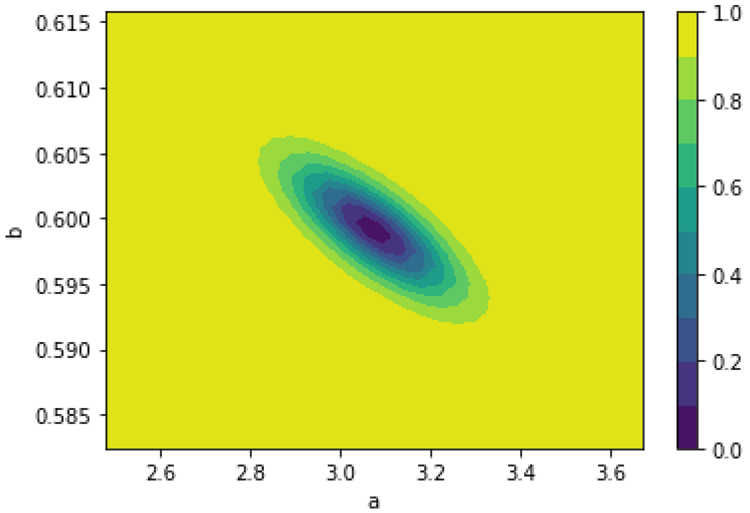
In [225]:

```
plt.plot(y, residual(out2.params)+y,'.' )
plt.show()
```



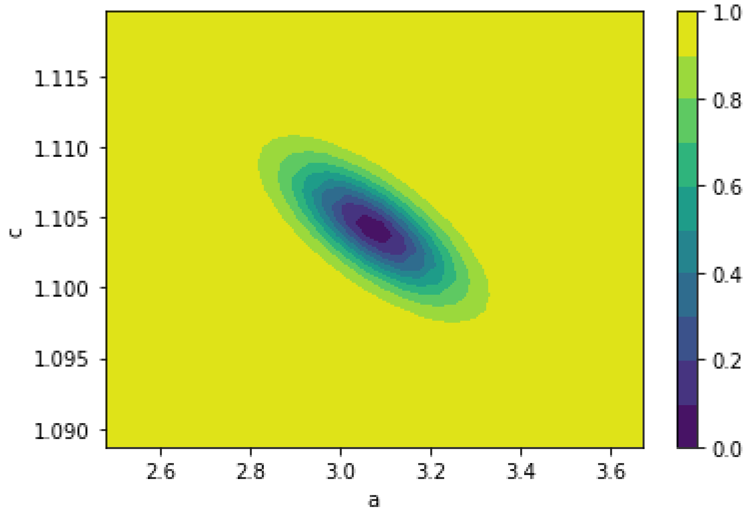
In [226]:

```
cx, cy, grid = lmfit.conf_interval2d(mini, out2, 'a','b',30,30)
plt.contourf(cx, cy, grid, np.linspace(0,1,11))
plt.xlabel('a')
plt.colorbar()
plt.ylabel('b')
plt.show()
```



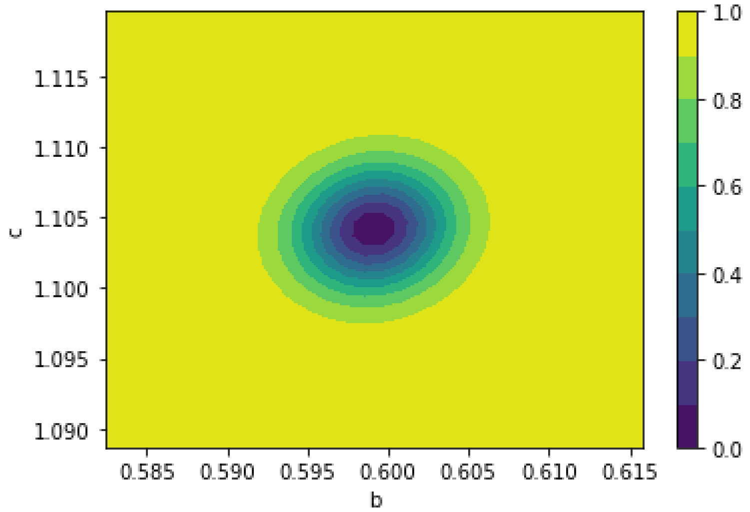
In [229]:

```
cx, cy, grid = lmfit.conf_interval2d(mini, out2, 'a','c',30,30)
plt.contourf(cx, cy, grid, np.linspace(0,1,11))
plt.xlabel('a')
plt.colorbar()
plt.ylabel('c')
plt.show()
```



In [234]:

```
cx, cy, grid = lmfit.conf_interval2d(mini, out2, 'b','c',30,30)
plt.contourf(cx, cy, grid, np.linspace(0,1,11))
plt.xlabel('b')
plt.colorbar()
plt.ylabel('c')
plt.show()
```



In [39]:

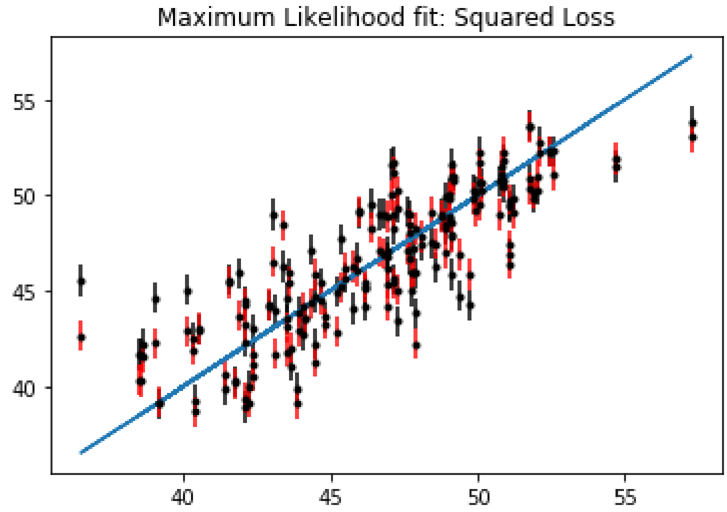
```
%matplotlib inline
#test square loss

import matplotlib.pyplot as plt
import numpy as np
x=np.random.rand(2,100)*10+20
y0=3+0.6*x[0]+1.1*x[1]
y=3+0.6*x[0]+1.1*x[1]+np.random.normal(0,1,100)*2
err=0.8

def squared_loss(theta, x=x, y=y, sigma_r=0.3,sigma_x=0.3,sigma_m=0.3):
    dy = y - theta[0] - theta[1] * x[0]-theta[2]*x[1]
    deno=sigma_r**2+x[0]**2*sigma_x**2+x[1]**2*sigma_m**2
    return np.sum(dy**2/deno**2)

def squared_loss_only_y(theta, x=x, y=y, sigma_r=0.3):
    dy = y - theta[0] - theta[1] * x[0]-theta[2]*x[1]
    return np.sum(dy**2/sigma_r**2)
theta1 = optimize.fmin(squared_loss_only_y, [0,0,0], disp=False)

plt.plot(y, y)
plt.errorbar(y,theta[0] + theta[1] * x[0]+theta[2]*x[1],err, fmt='.k', ecolor='blacl
plt.errorbar(y,theta[0] + theta1[1] * x[0]+theta1[2]*x[1],err, fmt='.k', ecolor='rec
plt.title('Maximum Likelihood fit: Squared Loss');
```



In [40]:

```
theta,theta1
```

Out[40]:

```
(array([-0.53597482,  1.02891375,  0.83023827]),
 array([-0.41875304,  0.71172023,  1.13522538]))
```

In [ ]:

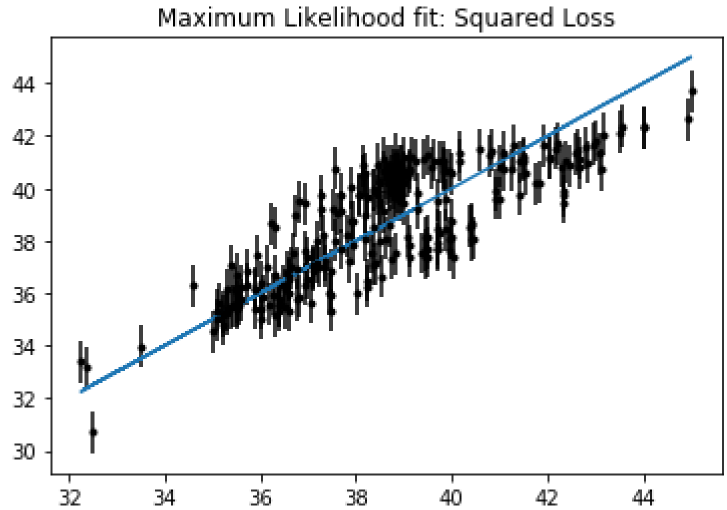
```

```

In [41]:

```
from scipy import optimize
x0=agn[:,1:3]
y=agn[:,0]
x=np.transpose(x0)

def squared_loss(theta, x=x, y=y, sigma_r=0.3,sigma_x=0.3,sigma_m=0.3):
    dy = y - theta[0] - theta[1] * x[0]-theta[2]*x[1]
    deno=sigma_r**2+x[0]**2*sigma_x**2+x[1]**2*sigma_m**2
    return np.sum(dy**2/deno**2)
theta = optimize.fmin(squared_loss, [0,0,0], disp=False)
plt.plot(y, y)
plt.errorbar(y,theta[0] + theta[1] * x[0]+theta[2]*x[1],err, fmt='.k', ecolor='blacl
plt.title('Maximum Likelihood fit: Squared Loss');
```



In [42]:

```
x.shape,y.shape
```

Out[42]:

```
((2, 254), (254,))
```

In [43]:

```
theta
```

Out[43]:

```
array([-0.66136949,  0.9217987 ,  0.09705679])
```

In [ ]:

In [ ]:

In [ ]: